CBSE X Date :	**MT EDUCARE LTD.** SUBJECT : **MATHEMATICS** **QUEST - I (Semi Prelim I)** **MODEL ANSWER PAPER**	**SET C** Marks : 80 Time : 3 hrs.

Any method mathematically correct should be given full credit of marks.

SECTION - A
Question number 1 to 6 carry 1 marks each.

1. $\alpha + \beta = 4, \qquad \alpha\beta = 1$

Required polynomial is $x^2 - (\alpha + \beta)\, x + \alpha\beta$

$= x^2 - 4x + 1$ **[1]**

2. $\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore \quad \frac{AB}{BC} = \frac{PQ}{QR}$

$\therefore \quad \frac{PQ}{QR} = \frac{1}{3} \qquad \left[\because \frac{AB}{BC} = \frac{1}{3}\right]$ **[1]**

3. $2x^2 - 4x + 3 = 0$

Now, $b^2 - 4ac = (-4)^2 - 4(2)(3)$

$= 16 - 24$

$= -8$

$\therefore \quad b^2 - 4ac < 0$

$\therefore$ The roots are not real. **[1]**

OR

2 is the root of $kx^2 - 14x + 8 = 0$

$\therefore \quad k(2)^2 - 14(2) + 8 = 0$

$\therefore \quad 4k = 20$

$\therefore \quad k = 5$ **[1]**

4. Here, a = 10, d = –3, n = 30.

Now, $a_n = a + (n - 1)d$

$\therefore a_{30} = 10 + (30 - 1)(-3)$

$= 10 - 87$

$\therefore a_{30} = -77$ **[1]**

5. $2x - 3y = 5$

Substituting $x = 3$, $y = a$, we get

$2(3) - 3(a) = 5$

$\therefore -3a = 5 - 6$

$\therefore -3a = -1$

$\therefore a = \frac{1}{3}$ **[1]**

OR

5. $x - y = 2$

$x + y = 4$

on solving, we get $x = 3$ and $y = 1$

$\therefore$ a = 3 and b = 1 **[1]**

6. PQ and PR are tangents.

$\therefore \angle OQP = \angle ORP = 90°$

Now, $\angle OQP + \angle QOR + \angle ORP + \angle QPR = 360°$

$\therefore 90 + QOR + 90 + 46 = 360$

$\therefore \angle QOR = 134°$ **[1]**

SECTION - B

Question number 7 to 12 carry 2 marks each.

7. Proof :

In ΔADC and ΔBAC,

$\angle ADC = \angle BAC$ ....[Given]

$\angle ACD = \angle BCA$ ....[common angle]

$\therefore \Delta ADC \sim \Delta BAC$ ....[By AA criterion]

$\therefore \frac{CA}{CB} = \frac{CD}{CA}$ ....[corresponding sides of similar triangles]

$\therefore CA^2 = CD.CB$ **[2]**

OR

7. Proof :

AP = PB

$\therefore \quad \frac{AP}{PB} = 1 \qquad ....(i)$

$AQ = QC$

$\therefore \quad \frac{AQ}{QC} = 1 \ . \qquad ...(ii)$

$\therefore \quad \frac{AP}{PB} = \frac{AQ}{QC} \qquad$ ....[From (i) and (ii)]

$\therefore \quad PQ \parallel BC \qquad$ ....[Converse at Basic proportionality Theorem] **[2]**

OR

8. Here, a = –37, d = 4, n = 12.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore \quad S_{12} = \frac{12}{2}[2(-37) + (12 - 1)4]$$

$$= 6(-74 + 44)$$

$$\therefore \quad S_{12} = -180$$

[2]

9. $3x^2 + 5x - 2 = 0$

$$\therefore \quad x^2 + \frac{5x}{3} - \frac{-2}{3} = 0$$

$$\therefore \quad x^2 + \frac{5x}{3} = \frac{2}{3} \qquad ....(i)$$

Third term $= \left(\frac{1}{2} \times \frac{5}{3}\right)^2 = \frac{25}{36}$

Adding $\frac{25}{36}$ on both sides of equation (i), we get

$$x^2 + \frac{5x}{3} + \frac{25}{36} = \frac{2}{3} + \frac{25}{36}$$

$\therefore \quad \left(x+\frac{5}{6}\right)^2 = \frac{49}{36}$

$\therefore \quad x+\frac{5}{6} = \pm\frac{7}{6}$

$\therefore \quad x = \pm\frac{7}{6} - \frac{5}{6}$

$\therefore \quad x = \frac{2}{6}$ or $x = \frac{-12}{6}$

$\therefore \quad x = \frac{1}{3}$ or $x = -2$ **[2]**

10. $2x + 3y - 46 = 0$; $3x + 5y - 74 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -46$

$a_2 = 3$, $b_2 = 5$, $c_2 = -74$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\therefore \quad \frac{x}{(3)(-74)-5(-46)} = \frac{y}{(-46)(3)-(-74)(2)} = \frac{1}{2(5)-(3)(3)}$$

$$\therefore \quad \frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$$

$$\therefore \quad \frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

$\therefore \quad x = 8$ and $y = 10$ **[2]**

11. $3x^2 - 10x + 7$ is the given polynomial.

$\alpha + \beta = \frac{10}{3}$, $\alpha\beta = \frac{7}{3}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{10}{3}\right)^2 - 2\left(\frac{7}{3}\right)$

$$= \frac{100-42}{9}$$

$\therefore \quad \alpha^2 + \beta^2 = \frac{58}{9}$ **[2]**

OR

11.

$$\begin{array}{r|l} & \quad x - 2 \\ \hline x+2 & \; 2x^2 + 3x + 1 \\ & \; 2x^2 + 4x^2 \\ & \; - \quad\;\; - \\ \hline & \qquad -x + 1 \\ & \qquad -x - 1 \\ & \qquad + \quad\; + \\ \hline & \qquad\qquad 3 \end{array}$$

Quotient = $2x - 1$

Remaindor = 3

12. Let AF = AE = a

BF = BD = b

CD = CE = c

Now, AB = AC

a + b = a + c

b = c

BD = CD

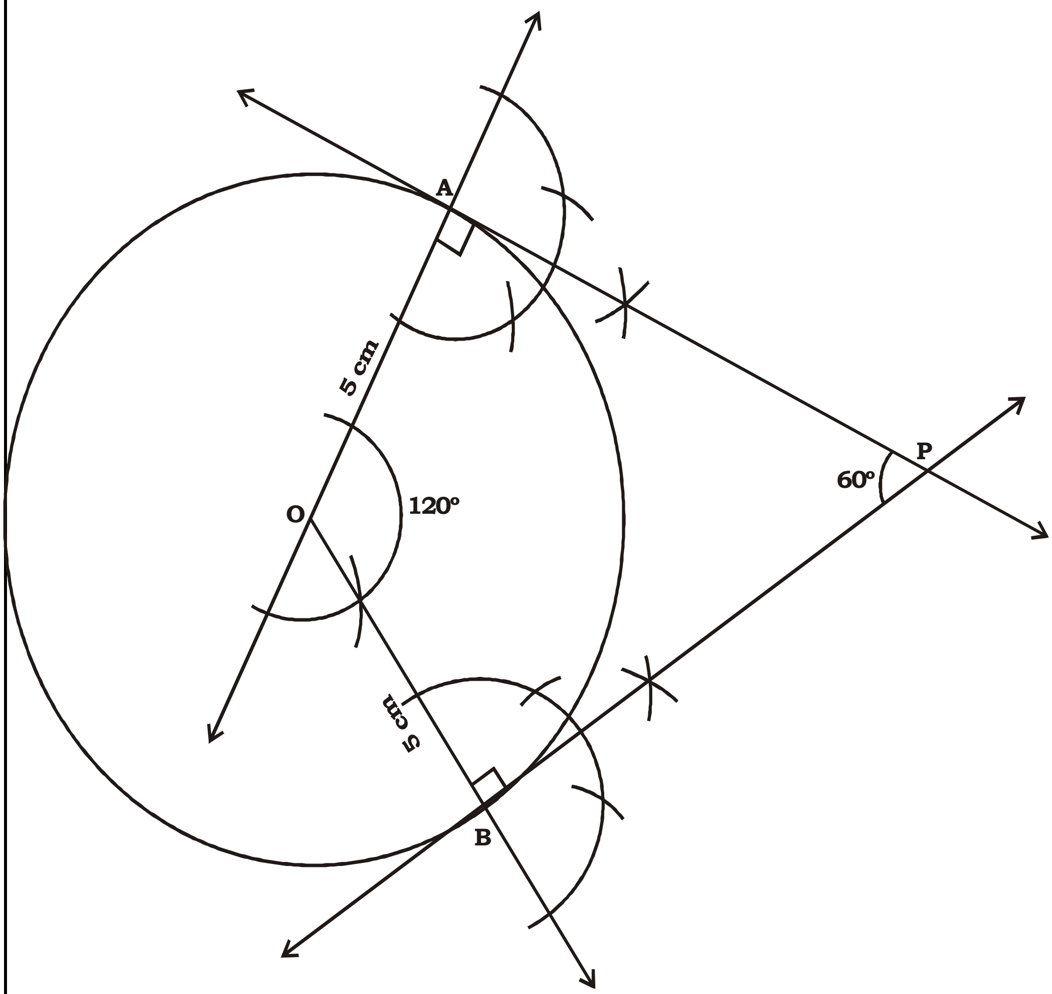
} Length of tangents from on external point to a circle are equal.

[2]

SECTION - C

Question numbers 13 to 22 carry 3 marks each.

13.



[3]

14. Here, a = 6, d = 6, n = 40

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{40} = \frac{40}{2}[2 \times 6 + (40 - 1) \times 6]$$

$= 20\ (12 + 234)$

$= 20 \times 246$

$S_{40} = 4920$ [3]

15.	$\frac{16}{x} - 1 = \frac{15}{x+1}$	
	y $\frac{16-x}{x} = \frac{15}{x+1}$	
	y $16x + 16 - x^2 - x = 15x$	
	y $x^2 = 16$	
	y $x = \pm 4$	**[3]**
	OR	
	Let x, $x + 2$ be the consecutive positive odd integers.	
	New, $x^2 + (x + 2)^2 = 290$	
	$\therefore$ $x^2 + x^2 + 4x + 4 = 290$	
	$\therefore$ $2x^2 + 4x - 286 = 0$	
	y $x^2 + 2x - 143 = 0$	
	y $x^2 + 13x - 11x - 143 = 0$	
	y $(x + 13)(x - 11) = 0$	
	y $x = -13$ or $x = 11$	
	As x is positive, so $x = 11$	
	y $x + 2 = 11 + 2 = 13$	**[3]**
16.	Let AB and AC represents the height of pole and length of wire respective.	
	$AC^2 = AB^2 + BC^2$	
	y $(24)^2 = (18)^2 + BC^2$	
	y $BC^2 = 576 - 324 = 252$	
	y $BC = 6\sqrt{7}$ m	
	y The stake should be driver $6\sqrt{7}$ from the base of the pole.	**[3]**
17.	Given : O is the centre of the circle.	
	PA and PB are tangerts.	
	To prove : PA = PB	
	Proof : In ΔOAP and ΔOBP,	
	$\angle QAP$, $\angle QBP = 90°$	
	QP = OP	
	OA = OB	
	y $\Delta OAP \cong \Delta OBP$ (RHS congruence criterion)	
	y PA = PB (c.p.c.t)	**[3]**

18. $\frac{2}{x}+\frac{3}{y} = 13; \frac{5}{x}+\frac{4}{y} = -2$

Let $\frac{1}{x} = a, \frac{1}{y} = b$

$\therefore$ 2a + 3b = 13; 5a – 4b = –2

on solving we get,
a = 2 and b = 3

$\therefore$ on resubstituting $\frac{1}{x} = a, \frac{1}{y} = b$

we get $x = \frac{1}{2}$ and $y = \frac{1}{3}$ **[3]**

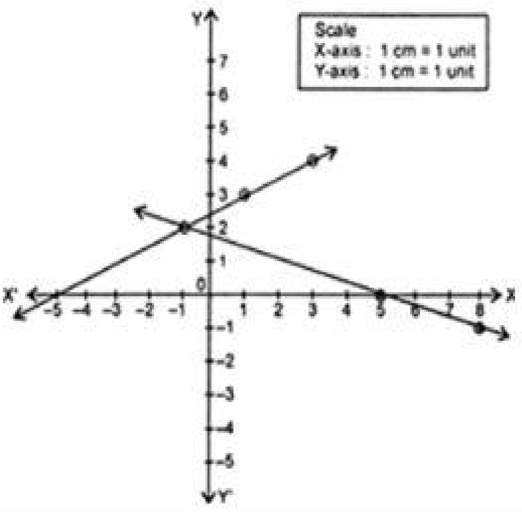
OR

We have,
$3x + y - 5 = 0$
$\Rightarrow y = 5 - 3x$
Thus we have following table :

x	5	–1	8
y	0	2	–1

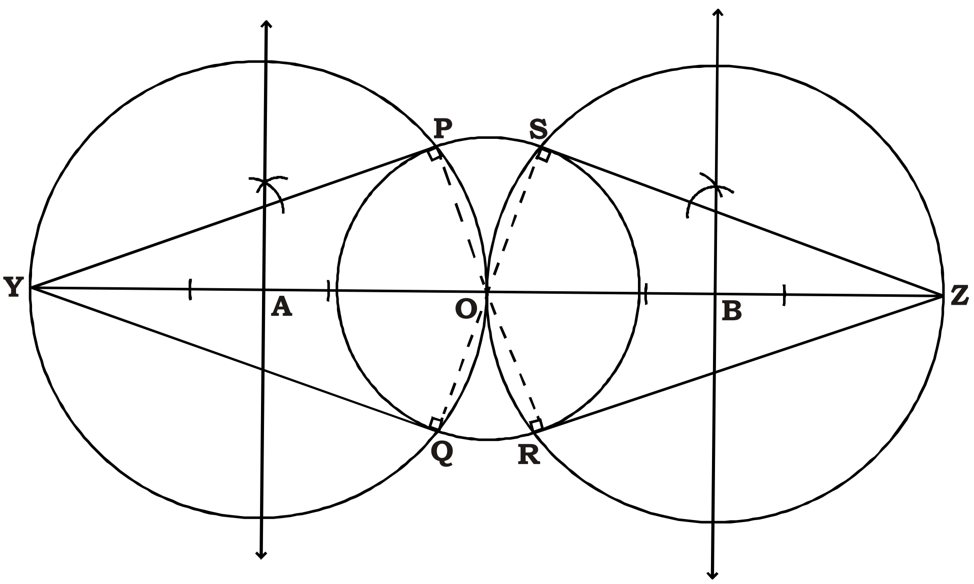
We have, $2x - y - 5 = 0$
$\Rightarrow y = 2x - 5$
Thus, we have following table :

x	–1	1	3
y	2	3	4



[3]

19.



[3]

20. Here, a = 5, l = 45, S_n = 400

Now, $S_n = \frac{n}{2}(a + l)$

y $400 = \frac{n}{2}(5 + 45)$

y $n = \frac{800}{50} = 16$

$l = a + (n - 1)d$

$45 = 5 + (16 - 1)d$

$15d = 40$

$d = \frac{8}{3}$ **[3]**

OR

20. Class I will plant 1 × 3 = 3 trees.

Class II will plant 2 × 3 = 6 trees.

Class III will plant 3 × 3 = 9 trees.

Class XII will plant 12 × 3 = 36 trees

Here, a = 3, d = 3, n = 12

Now, a = 3, d = 3, n = 12

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2 \times 3 + (12 - 11)3]$

$\therefore \quad S_{12} = 6(39) = 234$

234 trees will be planted. **[3]**

21. O is the centre of the concentric circle

AB is chord of larger circle and tangent at T for smaller circle.

$\angle OTA = 90^\circ$

$OA^2 = OT^2 + AT^2$

$\therefore \quad (5)^2 = (3)^2 + AT^2$

$\therefore \quad AT^2 = 16$

$\therefore \quad AT = 4$

Now, $AB = 2AT$ (The perpendicular drawn from the centre of the circle to the chord bisects the chord)

$= 2 \times 4$

$= 8cm$ **[3]**

22. Proof :

ΔABE] ΔACD

y $\quad AD = AE$

$AB = AC$

y $\quad \frac{AD}{AB} = \frac{AE}{AC}$ } (c.p.ct)

Also, $\angle A = \angle A$

$\Delta ADE \sim \Delta ABC$ (SAS criterion) **[3]**

OR

22. Proof:

$\angle AEP = \angle CDP = 90^\circ$

$\angle APE = \angle CPD$ (Vertically opposite angles)

$\therefore \quad \Delta AEP \sim \Delta CDP$ (AA Criterion)

$\angle B = \angle B$

$\angle ADB = \angle CEB = 90^\circ$

$\Delta ABD \sim \Delta ADB$

$\angle A = \angle A$

$\angle AEP = \angle ADB = 90^\circ$

$\Delta AEP \sim \Delta ADB$ (AA criterion) **[3]**

SECTION - D

Question numbers 23 to 30 carry 4 marks each.

23. Construction : Draw AD $\perp$ BC at D.

$AB = BC = AC = 6cm$

$PC = \frac{1}{3} BC = 2cm$

$\Delta ADC \cong \Delta ADB$ (By RHS criteruon)

$\therefore \quad DC = DB = \frac{1}{2} BC$ (c.p.ct)

$\therefore \quad DC = DB = 3cm$

Now, $PD = DC - PC = 3 - 2 = 1$ cm

$\therefore \quad AD^2 = AC^2 - CD^2$ (Pythagoras Theorem)

$= 6^2 - 3^2$

$\therefore \quad AD^2 = 27$

$AP^2 = AD^2 + PD^2$

$= 27 + 1$

$\therefore \quad AP^2 = 28$

$\therefore \quad AP = 2\sqrt{7}$ cm **[4]**

OR

Given : ΔABC, $\angle ABC = 90°$

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw BD $\perp$ AC at D.

Proof :

$\Delta ABC \sim \Delta ADB$	$\Delta ABC \sim \Delta BDC$
$\therefore \quad \frac{AB}{AD} = \frac{AC}{AB}$	$\therefore \quad \frac{AC}{BC} = \frac{BC}{DC}$
$\therefore \quad AB^2 = AC.AD$ ....(i)	$\therefore \quad BC^2 = AC . DC$ ....(ii)

Adding (i) and (ii), we get

$AB^2 + BC^2 = AC.AD + AC.DC$

$AB^2 + BC^2 = AC(AD + DC)$

$AB^2 + BC^2 = AC^2$ **[4]**

24. $\frac{10}{x+y}+\frac{2}{x-y} = 4; \quad \frac{15}{x+y}-\frac{5}{x-y} = -2;$

Let $\frac{1}{x+y}$ = a and $\frac{1}{x-y}$ = b

$\therefore$ 10a + 2b = 4 and 15a – 5b = –2

on solving both equations, we get

a = $\frac{1}{5}$ and b = 1

Resubstituting a = $\frac{1}{x+y}$ and b = $\frac{1}{x-y}$, we get

$x + y = 5$ and $x - y = 1$

on solving, we get $x = 3$ and $y = 2$ **[4]**

OR

ABCD is cyclic

$\therefore \quad \angle A + \angle C = 180°$

$\therefore \quad 6x + 10 + x + y = 180$

$\therefore \quad 7x + y = 170 \quad .....(i)$

Also, $\angle B + \angle D = 180°$

$\therefore \quad 5x + 3y - 10 = 180$

$\therefore \quad 5x + 3y = 190 \quad ......(ii)$

on solving equations (i) and (ii), we get

$x = 20$ and $y = 30$

$\angle A = 6x + 10 = (6 \times 20) + 10 = 130°$

$\angle B = 5x = 5 \times 20 = 100°$

$\angle C = x + y = 20 + 30 = 50°$

$\angle D = 3y - 10 = 3(30) - 10 = 80°$ **[4]**

OR

25. $\frac{1}{x+4}-\frac{1}{x-7} = \frac{11}{30}$

$\therefore \quad \frac{(x-7)-(x+4)}{x^2-3x-28} = \frac{11}{30}$

$\therefore \quad \frac{-11}{x^2 - 3x - 28} = \frac{11}{30}$

$\therefore \quad -30 = x^2 - 3x - 28$

$\therefore \quad x^2 - 3x + 2 = 0$

$\therefore \quad x - 2x - x + 2 = 0$

$\therefore \quad (x - 2)(x - 1) = 0$

$\therefore \quad x = 2 \text{ or } x = 1$ **[4]**

OR

25. Let the actual speed of the train be x.
New speed = $x + 5$

Now, $\frac{300}{x+5} - \frac{300}{x} = -2$

$\therefore \quad \frac{x - x - 5}{x^2 + 5x} = \frac{-2}{300}$

$\therefore \quad \frac{-5}{x^2 + 5x} = \frac{-1}{150}$

$\therefore \quad 750 = x^2 + 5x$

$\therefore \quad x^2 + 5x - 750 = 0$

$\therefore \quad x^2 + 30x - 25x - 750 = 0$

$\therefore \quad (x + 30)(x - 25) = 0$

$x = -30 \text{ or } x = 25$

As speed cannot benegative, so $x = 25$

$\therefore$ The actual speed of the train is 25 km. **[4]**

26.	$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$ $x(3x - 4) + 1\ (3x - 4)$ $(x + 1)\ (3x - 4)$ Get zeros, $(x + 1)(3x - 4) = 0$ y $\quad x = -1$ and $x = \frac{4}{3}$ Let $\alpha = -1$ and $= \frac{4}{3}$ $\alpha + \beta = -1 + \frac{4}{3} = \frac{1}{3}$ $\alpha\beta = -1 \times \frac{4}{3} = \frac{-4}{3}$ $\frac{-b}{a} = \frac{-(-1)}{3} = \frac{1}{3}$ $\frac{c}{a} = \frac{-4}{3}$ y $\quad \alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2$ ò 13 $= \left(\frac{1}{3}\right)^2 - 2\left(\frac{-4}{3}\right)$ $= \frac{1+24}{9}$ $= \frac{25}{9}$ Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{25}{9} \div \left(\frac{-4}{3}\right) = \frac{-25}{12}$	[4]
27.	$a_4 + a_8 = 24$ $\therefore\ a + 3d + a + 7d = 24$ $\therefore\ 2a + 10d = 24$ $\therefore\ a + 5d = 12$ ....(i) $a_6 + a_{10} = 44$ $\therefore\ a + 5d + a + 9d = 44$ $\therefore\ 2a + 14d = 44$	

$\therefore$ a + 7d = 22 ....(ii)

on solving equations (i) and (ii), we get

d = 5 and a = –13

Now, $_{10}$ =

$\frac{10}{2}\left[2(-13)+(10-1)5\right]$

= 5(–26 + 45)

$\therefore$ S_{10} = 95 **[4]**

28. Construction : Join O with point of contacts P, Q, R and S.

Also, join O to A, B, C and D.

Proof :

In ΔAOS and ΔAOP,

$\angle ASO = \Delta APO = 90°$

OA = OA

OS = OP

$\therefore$ $\Delta AOS \cong \Delta AOP$ (By RHS criterion)

$\therefore$ $\angle AOS = \angle AOP$ = a(c.p.c.t)

similarly, $\angle BOP = \angle BOQ = b$,

$\angle COQ = \angle COP = C$,

Now, a + a + b + b + c + c + d + d = 360

$\therefore$ 2a + 2b + 2c + 2d = 360

$\therefore$ a + b + c + d = 180

$\therefore$ (a + b) + (c + d) = 180

$\therefore$ $\angle AOB + \angle COD = 180°$

Also, (b + c) + (a + d) = 180

$\therefore$ $\angle BOC + \angle AOD = 180°$ **[4]**

30. Let x be the marks of shrishti in mathematics.

Then her marks in English is $(30 - x)$.

Now, $(x + 2)(30 - x - 3) = 201$

$\therefore$ $(x + 2)(27 - x) = 210$

$\therefore \quad 27x - x^2 + 54 - 2x = 210$

$\therefore \quad x^2 - 25x + 156 = 0$

$\therefore \quad x^2 - 13x - 12x + 156 = 0$

$\therefore \quad (x - 13)(x - 12) = 0$

$\therefore \quad x = 13$ or $x = 12$

If $x = 13$ then $30 - x = 17$

If $x = 12$ then $30 - x = 18$

The marks of shrishti are :

13 in MAthematics and 17 English or

12 in mathematics and 18 in English. [4]

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